Calculating Required Nose Weight

Rocket Science 101, Chapter 9

Reprinted from August 1999 Tripoli Central California News

By Bob Dahlquist

Calculating how much nose weight to add is easy when you understand the basic concept. If you played on a see-saw as a child, you have already made use of this concept, without the math. The moment of the rocket as it is must balance the moment of the added nose weight at the desired new balance point. Moment is the torque tending to tilt the seesaw or rocket one way or the other. On the seesaw, the moment of each kid is equal to his weight multiplied by his distance from the pivot. When the moments are equal, the seesaw is in balance.

In algebraic notation, the concept looks like this:

$W_n X_n = W_r X_r$

Where

W_n = Nose weight to be added

X_n = Distance of nose weight's CG from the desired new rocket CG

W_r = Existing rocket liftoff weight

X_r = Distance of existing rocket CG from the desired new CG (in other words, the distance you want to move the CG forward)

$W_n X_n$ = Nose weight moment

$W_r X_r$ = Existing rocket CG moment

For those who aren't into algebra, there is an easy worksheet below. I'll also explain the formula above:

Moment is another name for torque or potential torque. It is the weight of a particular item, multiplied by its distance from a particular point. This distance is called the "moment arm". In this case, the reference point is the desired balance point, or new CG.

The moment of the rocket ($W_r X_r$), before adding more nose weight, is equal to the weight of the rocket, multiplied by the distance from the existing CG to the desired new CG.

The moment of the nose weight or ballast ($W_n X_n$) is equal to its weight, multiplied by the distance between the nose weight's center of mass and the desired new CG.

Solving for W_n gives:

$W_n = W_r (X_r/X_n)$

Note that the nose weight required is inversely proportional to the ratio of distances.

$W_n/W_r = X_r/X_n$

* * *

Easy Nose Weight Work Sheet for those who aren't into Algebra:

All measurements must be in the same units. For example, length measurements must be all inches or all cm; do not mix feet with inches. Weights must be all ounces or all grams; do not mix pounds and ounces.

1. X_r, distance from existing CG to desired CG: ________________________

2. X_n, distance from nose weight CG to desired CG: ________________________

3. (X_r/X_n), divide #1 by #2 and put the answer here: _______________________

Note that this number should be less than one.

4. W_r, liftoff weight of the rocket as it is: ___________________________________

5. Multiply #4 by #3. The answer is the amount of nose weight (W_n) to add: ___________________________________

6. Add #5 to #4 to get the new liftoff weight. ___________________________

Make sure the rocket will not be too heavy for the motor. A minimum thrust-to-weight ratio of 5 to 1 is recommended on calm days; 10 to 1 on windy days. The rocket's weight must not exceed the motor manufacturer's maximum recommended liftoff weight for the motor used.

* * *

Extra Work Sheet for a Second Calculation:

1. X_r, distance from existing CG to desired CG: ________________________

2. X_n, distance from nose weight CG to desired CG: ________________________

3. (X_r/X_n), divide #1 by #2 and put the answer here: _______________________

Note that this number should be less than one.

4. W_r, liftoff weight of the rocket as it is: ___________________________________

5. Multiply #4 by #3. The answer is the amount of nose weight (W_n) to add: ___________________________________

6. Add #5 to #4 to get the new liftoff weight. ___________________________

Make sure the rocket will not be too heavy for the motor. . A minimum thrust-to-weight ratio of 5 to 1 is recommended on calm days; 10 to 1 on windy days. The rocket's weight must not exceed the motor manufacturer's maximum recommended liftoff weight for the motor used.

Predicting Your Rocket's CG

Before Buying the Motor (know before you buy):

Being able to predict the CG of your rocket is handy when you're building a new rocket, or you're considering flying your rocket with a larger motor. Here's how it works.

At any arbitrarily chosen reference point, the moment of the fully loaded rocket will equal the combined moments of the motor and the rest of the rocket (airframe and payload).

$$W_rX_r = W_mX_m + W_aX_a$$

Subscripts: r refers to the fully loaded rocket, a refers to the rocket without the motor (airframe and payload), and m refers to the motor.

X is the distance of the center of mass or CG of each item from the datum or reference point. All measurements must be in the same units and from the same reference point or datum. You can use whatever reference point you want, as long as you take all the x measurements from that same point. Usually we use the tip of the nose.

You can assume that the CG of the motor is in the center of the tubular casing. X_m is then the distance of that point from the tip of the nose (or other datum point).

W is the weight of each item (or mass, if you wish). All must be in the same units. W_m is the weight of the motor with its propellant; you can get it from the manufacturer or his catalog.

W_r is the liftoff weight of the entire rocket.

X_r is the CG of the rocket with the new motor in it, the quantity we want to find. Solving the above equation for X_r gives:

$$X_r = (W_mX_m + W_aX_a) \div W_r$$

Remember that the liftoff weight includes the weight of the motor with its propellant.

$W_r = W_m + W_a$

Substituting, we get:

$$X_r = (W_m X_m + W_a X_a) \div (Wm + W_a)$$

* * *

Reprinted from August 1999 Tripoli Central California News

Easy CG Prediction Work Sheet

for those who aren't into algebra:

1. Length of the motor casing (not including forward & aft closures): ________________________________

2. Divide motor casing length by 2: ________________________________

3. Length of Rocket from tip of nose to aft bulkhead (what the motor's aft closure thrusts against while the motor is burning): ________________________

4. X_m, location of the motor's CG when installed in the rocket: Subtract #2 from #3: _____________________________

5. W_m, weight of the motor, including propellant: ______________________

6. $W_m X_m$, the motor moment: Multiply #4 by #5: _______________________

7. W_a, weight of the rocket with everything in it except the motor: _______________________________

8. X_a, balance point of the rocket with everything in it except the motor (measured from the tip of the nose): _______________________________

9. $W_a X_a$, the airframe moment: Multiply #7 by #8: _______________________

10. $(W_m + W_a)$, liftoff weight: Add #5 and #7: ________________________

11. Total Moment: Add #6 and #9: _______________________________

12. **Rocket CG with motor installed:** Divide #11 by #10: _______________

NOTE: Steps 1 through 4 are designed for Aerotech and Dr. Rocket motors. For other brands, such as Kosdon motors, figure out item 4, X_m, in a way that works for that type of motor. Or get the information from the manufacturer.

* * *

Reprinted from August 1999 Tripoli Central California News

WIND-CAUSED INSTABILITY

By Bob Dahlquist
TRA #3032

At our January launch, two rockets (a LOC/Precision Hi-Tech H45™ and a Binder Design Thug), built from kits and supposedly stable, went unstable when launched, looping downwind and crashing. An Estes Phoenix and R2D2 surprised us by cruising downwind rather than upwind. Wind velocity at the time was between 10 and 20 miles per hour.

Some 32 other rockets were launched in wind velocities between 10 and 20 miles per hour, and flew normally.

Why did four rockets go unstable, even though they were built from kits and presumably had normal margins of stability? And why did the 32 other rockets fly normally on the same day?

Understanding what the margin of stability actually means, and its limits, is the key to understanding what happened to these four rockets, and preventing the same problem from occurring at future launches.

First of all, it is important to remember that a rocket's center of pressure is not fixed. The Barrowman formula (and software based on it) gives the center of pressure at *zero angle of attack*; the Center of Lateral Area method gives a rough, conservative estimate of the center of pressure at *90 degrees angle of attack*. At angles between zero and 90 degrees, the actual center of pressure will be somewhere between these two locations.

As the angle of attack increases from zero toward 90 degrees, the center of pressure moves forward (Fig. 1). If the center of pressure moves forward beyond the center of gravity, the rocket becomes unstable and veers off.

The stability margin, as traditionally defined in High Power rocketry, is the distance between the center of gravity and the center of pressure at zero angle of attack. It is the distance through which the center of pressure can move forward before the rocket goes unstable as the angle of attack increases. This margin is customarily measured in multiples of the body tube diameter, or *calibers*.

For each rocket, there is a corresponding maximum angle through which the angle of attack can vary without the rocket going unstable. This maximum angle is the angle of attack at which the center of pressure coincides with the center of gravity. Thus, the stability margin can also be specified as the number of *degrees* that the angle of attack can vary to the right or left before the rocket goes unstable. (I'll call this the *angular stability margin*.)

The angle of attack is the same as the angle of the wind relative to the rocket's axis. On a dead calm day, with a perfect rocket, the relative wind is produced by the motion of the rocket alone, and is parallel to the rocket's axis; the angle of attack is zero.

Crosswind and the Angle of Attack

When there is a crosswind, it combines with the rocket's velocity by vector addition to produce a resultant relative wind. The velocity of the crosswind and the velocity of the relative wind caused by the rocket's motion can each be represented by an arrow, of a length proportional to the velocity it represents.

To add the vectors graphically, the arrows are drawn at angles corresponding to their respective wind directions, with the head of one arrow touching the tail end of the next. The resultant, or vector sum, is then represented by a new arrow drawn from the tail end of the first vector to the head end of the last vector (Fig. 2).

The resultant can also be calculated through the use of trigonometry or analytical geometry.

If the launcher is not tilted, and the rocket is parallel to the launch rod, the trigonometry is very simple, since the crosswind is at right angles to the rocket's velocity. Any scientific calculator can calculate the angle of relative wind directly.

Relative wind angle = Angle of attack = Tan-1 (Vw/Vr) (Equation 1)

where Vw = Crosswind velocity, ft./sec.

Vr = Rocket velocity, ft./sec.

First calculate the ratio of velocities, and then two more keystrokes give you the angle.

For example, if the rocket velocity is equal to the crosswind velocity, the ratio is 1, and the angle of attack is 45 degrees.

If the rocket velocity is twice the crosswind velocity, the ratio is 0.5, and the angle of attack is 26.6 degrees (rounded off to the nearest tenth).

How the Rocket is Affected

As the rocket leaves the top of the launch rod, if the angle of attack is greater than the rocket's angular stability margin, the rocket will go unstable.

If it has a large polar moment of inertia and good acceleration, the rocket may develop enough speed to bring the relative wind back within its stability margin before veering off very much.

If it has a small polar moment of inertia, high damping, and low acceleration, the rocket is likely to turn straight downwind and then fly level like a cruise missile. (When such a rocket's angular stability margin is *greater* than the relative wind angle, it is likely to weathercock and cruise *upwind*.)

If it has a large polar moment of inertia combined with poor acceleration and poor damping, the rocket may loop and crash before ever gaining enough speed to restore stability. Once it starts turning downwind, its rotational momentum tends to keep it turning.

To prevent dangerous flights such as these, we need a way to determine, for any given crosswind, how much thrust a rocket must have so that it can reach the velocity needed for stability.

Calculating the Minimum Velocity for Stability

The velocity needed is that velocity which results in an angle of relative wind, or angle of attack, that is within the angular stability margin of the rocket.

Thus, to be able to calculate the required velocity, we must first either measure or estimate the angular stability margin of the particular rocket.

If the stability margin of a particular rocket can not be determined easily, then a worst-case conversion factor can be used, based on tests of numerous other rockets. For example, based on the test results shown in Figure 1, you could multiply the stability margin in calibers by 13 to get a worst-case stability margin in degrees. (This conversion factor should be considered preliminary, as it is based on tests of only 4 rockets.) Note that some rockets will have more than 13 degrees per caliber of stability; for example, the Alpha II has 26 degrees at one caliber, and 39 degrees at 1.5 caliber stability margin.

In a wind tunnel or in a large, open field (such as a launch site) on a windy day, a rocket can be mounted on a pivot at its balance point, and its actual angular stability margin can be measured directly if the rocket is not too large.

Or, with the proper mathematical model, it should be possible to predict the angular stability margin before even building the rocket.

Calculating the Desired Thrust/Weight Ratio

Once we know the stability margin in degrees, the velocity needed can be calculated by solving equation 1 (above) for Vr:

Vr = Vw/Tangent of Stability Margin Angle (Equation 2)

For example, suppose the stability margin of a particular rocket is 36 degrees. Let's assume the wind is blowing at 15 miles per hour. Multiply 15 by the conversion factor, 22/15, because we need the crosswind velocity in ft./sec., rather than mph. The result is 22 ft./sec. Now divide by the tangent of 36 degrees (0.7265) to get the required rocket velocity leaving the launcher, which is 30.28 ft./sec.

The acceleration required to produce a given velocity depends on the length of the launch rod, rail, or tower.

a = vr2/2h (Equation 3)

Where a = acceleration required, in ft/sec.2

vr = required velocity for stability, in ft/sec.

h = height of launch rod, rail, or tower, in ft.,

measured from the bottom of the launch lug

(or the forward launch lug if there are two)

(or the center of mass if there are no lugs)

to the top of the launcher.

(Note that if the rocket has two launch lugs spaced far apart, this fact alone can cause the nose to be blown downwind on a windy day when the upper lug clears the launcher, freeing the front end, while the tail end is still constrained.)

Once we know the required acceleration, it is easily converted to a thrust-to-weight ratio by dividing by 32.17 and then adding 1.

Table 1 gives thrust-to-weight ratios required for complete stability at various wind speeds, and 1 or 2 calibers of stability margin, assuming a worst-case conversion factor of 13 degrees per caliber.

Comparison of our January flight card data with Table 1 indicated that many of the 32 rockets that had more or less normal flight paths must have had more than 13 degrees of angular stability margin, and/or sufficient moment of inertia on their yaw axes to prevent them bearing off too far while accelerating to the minimum velocity.

All rockets that had at least a 10:1 thrust-to-weight ratio flew normally despite the wind. But keep in mind that most of the rockets that day were flown in winds of 16 mph or less.

Table 1

Crosswind		Min.Vr	Minimum Thrust to Weight Ratio				
MPH	Ft/Sec	Ft/Sec	3'Rod	4'Rod	5'Rod	8'Rod	20'Twr
5	7.33	31.8*	7.3	6.2	5.5	3.9	2.0
5	7.33	15.0**	2.4	2.2	2.0	1.6	1.2
10	14.67	63.5*	26.1	21.9	18.9	11.4	4.9
10	14.67	30.1**	6.6	5.7	5.0	3.6	1.9
15	22.00	95.3*	57.5	48.0	41.3	26.7	9.8

15 22.00 45.1** 13.6 11.5 10.0 6.7 3.0

20 29.33 127* 101.3 84.6 72.6 46.6 16.7

20 29.33 60.1** 22.5 19.7 17.0 11.2 4.5

* 13 degrees or one caliber stability margin

** 26 degrees or two calibers stability margin

Note: This table assumes the following heights for the launch lug above the bottom of the rod:

3' rod: 6" 4' rod: 1 ft. 5' rod: 1.5 ft. 8' rod: 2.5 ft.

20' tower: Rocket CG 4 ft. above the tower base.

Low Thrust to Weight Ratios are Dangerous

Three of the rockets that went unstable or cruised downwind had thrust-to-weight ratios between 5.7:1 and 6.5:1; less than nearly all the other rockets launched that day, according to flight card data. The Hi-Tech H45 was flown with a G40 motor, the Thug with an F25, and the Phoenix with a D12. The R2D2 had a thrust-to-weight ratio of about 8:1, but has a very low polar moment of inertia and probably a low stability margin also, because it's so short.

The acceleration of the three low-thrust rockets was only about 5 g's. A rocket's velocity at the top of a 3 foot launch rod, with uniform 5 g's acceleration, would be about 28 feet per second. With a crosswind velocity of 15 miles per hour (22 feet per second), the angle of relative wind would be about 38 degrees. Just before we closed down the launch, the wind velocity was about 20 miles per hour (29.33 ft./sec.) and the relative wind angle on these rockets would be about *46 degrees*. I doubt that most high power rockets would be stable at such a large angle of attack.

Taller Launchers are Preferred

A taller launcher gives the rocket time to accelerate to a higher velocity before it leaves the launcher; thus the relative wind angle will be smaller at any given crosswind speed. But launch rods can't be made longer without also being thicker, otherwise they will flex too much. Thicker rods require larger launch lugs, which have more drag. And unless the rocket has a built-in roll rate or spin, the unbalanced drag on one side will cause the rocket to curve toward the launch lug in flight.

Rail or tower launchers solve this problem. The rail or tower can be made as long as necessary for velocity, and as wide as necessary for stiffness, without increasing the drag of the rocket.

The length of the rod, rail, or tower required for a given thrust-to-weight ratio and velocity can be calculated by solving Equation 3 for h:

h = vr2/2a (Equation 4)

where

$a = 32.17\ [(f_i/w)-1]$

f_i/w = Thrust to weight ratio

f_i = Average initial thrust while the rocket is

accelerating up the launcher (pounds)

w = Weight of the rocket at launch (pounds)

vr = velocity required for stability, ft./sec.

The -1 is for gravity.

The average initial thrust used with Equations 3 and 4 is defined (by the author) as the thrust averaged over the period of time that the rocket is accelerating up the launch rod, rail, or tower. This period of time is the first 0.1 to 0.2 seconds of burn time, for typical model and high power rocket launchers (see thrust curves).

The average initial thrust is usually not the same as the average thrust (which is averaged over the entire burn time of the motor). The difference is most pronounced with thrust profiles like those of the Estes A10 and C5, the Aerotech/Apogee F10, and the Aerotech K125W. With thrust profiles like these, the average initial thrust is several times higher than the average thrust, to give the rocket good acceleration off the launcher.

There are some motors with initial thrust *lower* than their average thrust, such as the Aerotech K185W, the Rocketflite Silver Streaks, and the Plasmajet I102.

Most Aerotech rocket motors have an initial thrust somewhere between 1.1 and 2.0 times their average thrust.

Conclusions:

- Rocketeers intending to fly on windy days should learn more about their rocket's stability than merely whether or not the CG is at least 1 caliber forward of the CP.
- RSOs and LCOs on windy days should beware rockets with low thrust to weight ratios, especially when they are unusually wide for the motor used.
- High thrust/short burn time motors are preferred in windy conditions.
- Divide the average thrust of a typical Aerotech motor (in Newtons) by 3.42 to get a rough estimate of the *initial* thrust in pounds. In windy conditions, divide by 30, or divide by 100 and then multiply by 3 for a rough, usually conservative estimate of maximum rocket gross weight for a thrust/weight ratio of about 10 or 11 to 1 at launch. (Based on initial thrust 1.3 times average thrust.) Just remember that not all Aerotech motors are typical. It's always more accurate to look at the thrust curve.
- On a *calm* day, you can divide the average thrust by 20, which gives the launch weight for a thrust/weight ratio around 5.85:1. As always, the rocket's liftoff weight should never exceed the maximum liftoff weight specified for the particular motor by its

manufacturer.

- In windy weather, a scale on the RSO table should be considered a necessity.
- Launch lugs spaced far apart are dangerous in windy weather. Single lugs at the CG are safer.
- It is not a good idea to launch rockets in winds between about 15 mph and the 20-mph limit, unless their stability characteristics are particularly favorable; for example, rockets with weighted noses, diameters not much greater than the motor diameter, and high thrust to weight ratios. (Be aware that these characteristics will cause the rocket to fly high, and drift far downwind, unless the burn time is short.)
- Towers and rail launchers are very much to be preferred over launch rods in windy weather.

* * *

Glossary:

Barrowman Formula: A complicated algebraic formula for calculating the center of pressure of a conventionally shaped rocket, using a moment method. This formula can be found in the back of the *Handbook of Model Rocketry, 6th Edition*, by George Harry Stine (1994). The Rogers Aeroscience CP program, and one or more others, are based on this formula.

Center of Mass, Center of Gravity, or CG: The point on the rocket at which its mass or weight balances; the balance point.

Center of Pressure (CP): The point on a rocket at which aerodynamic lateral forces (acting at 90 degrees to the rocket axis) balance at any given angle of attack.

Damping: The tendency to reduce the amplitude of any oscillations or overshoots, and return to equilibrium quickly. Rockets with large fins tend to have more damping.

Polar Moment of Inertia, or Rotational Inertia: The rotational equivalent of mass; the tendency of an object to resist any change in its rate of rotation around a particular axis (in this article, the yaw axis). Longer rockets and rockets with weighted noses have more rotational inertia.

Rotational Momentum: Rotational Inertia multiplied by the speed of rotation of an object.

Vector: A quantity that has both magnitude and direction in space; A line or arrow representing such a quantity.

Velocity: The combination of speed and direction.

* * *

February 4, 1998